10.6 Parametric Equations

Definitions/Characteristics

- Introduce a 3rd variable- called the parameter
- A common parameter used is time (t) or an angle (trig)
- (x, y) is the place, "t" is the time it is there (at that place)

Graphing Parametric Equations

- 2 OPTIONS:
  - (1) Use a chart to find rectangular points.

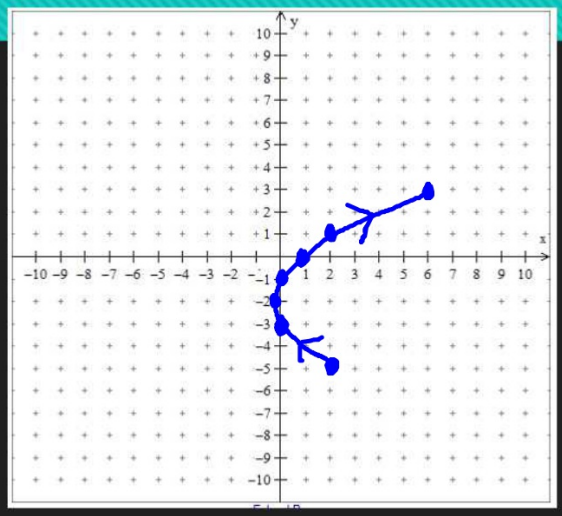
    Plot the points.

    Show the orientation (flow) by arrows
  - (2) Convert to a rectangular equation and graph it

Graphing Using a Chart

$x = t^2 + t$ $\qquad$ $y = 2t - 1$

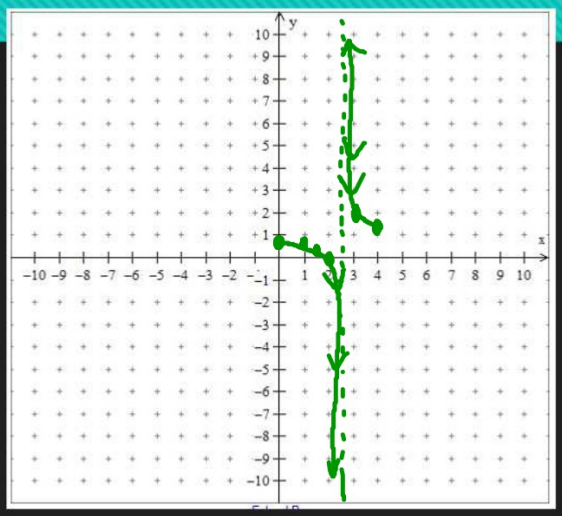
t	x	y
-2	2	-5
-1	0	-3
$-\frac{1}{2}$	$-\frac{1}{4}$	-2
0	0	-1
$\frac{1}{2}$	$\frac{3}{4}$	0
1	2	1
2	6	3



Graphing Using a Chart

$$x = t + 2 \qquad y = \frac{2t}{2t - 1}$$

t	x	y
-2	0	4/5
-1	1	2/3
$\frac{1}{2}$	1.5	1/2
0	2	0
$\frac{1}{2}$	2.5	U
1	3	2
2	4	4/3



Converting from Parametric to Rectangular (Eliminating the Parameter)

- Steps:
  1. Solve for the parameter
  2. Substitute the parameter into the 2nd equation
  3. Simplify

Examples: Parameter is time

1.) $x = t + 4,\ y = t - 7$

$x - 4 = t$

$y = x - 4 - 7$

$y = x - 11$

$x + 4 = t^2$

$\pm\sqrt{x+4} = t$

$y = \pm\sqrt{x+4} - 2$

2.) $x = t^2 - 4,\ y = t/2$

$2y = t$

$x = (2y)^2 - 4$

$x = 4y^2 - 4$

Examples: Parameter is time

3.) $x = \sqrt{t} + 5$, $y = 2t - 7$

$(x-5)^2 = (\sqrt{t})^2$

$(x-5)^2 = t$

$y = 2(x-5)^2 - 7$

4.) $x = |t - 2|$, $y = t + 4$

$y - 4 = t$

$x = |y - 4 - 2|$

$x = |y - 6|$

$x = y - 6$ | $x = -(y-6)$

$x + 6 = y$, $x \geq 0$ | $-x = y - 6$

$-x + 6 = y$, $x > 0$

Examples: Eliminating the Angle Parameter

1.) $x = 3cos\theta, y = 4sin\theta$

$\frac{x}{3} = cos\theta \quad \frac{y}{4} = sin\theta$

$cos^2\theta + sin^2\theta = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

$\boxed{\frac{x^2}{9} + \frac{y^2}{16} = 1}$

2.) $x = 2cos\theta, y = 2sin\theta$

$\frac{x}{2} = cos\theta \quad \frac{y}{2} = sin\theta$

$cos^2\theta + sin^2\theta = 1$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{4} + \frac{y^2}{4} = 1$

$\boxed{x^2 + y^2 = 4}$

Converting from Rectangular to Parametric

- Must be given the parameter
- Steps:
  1. Plug the given parameter into the rectangular equation
  2. Simplify

Examples

Given: $y = 1 - x^2$

Find the parametric equations given the following parameters:

1.) $t = x$

$y = 1 - t^2$

2.) $t = x + 3$

$t - 3 = x$

$y = 1 - (t-3)^2$

$y = 1 - (t^2 - 6t + 9)$

$y = 1 - t^2 + 6t - 9$

$y = -t^2 + 6t - 8$

3.) $t = 1 - x$

$t - 1 = -x$

$-t + 1 = x$

$y = 1 - (-t+1)^2$

$y = 1 - (t^2 - 2t + 1)$

$y = 1 - t^2 + 2t - 1$

$y = -t^2 + 2t$

Practice Problems

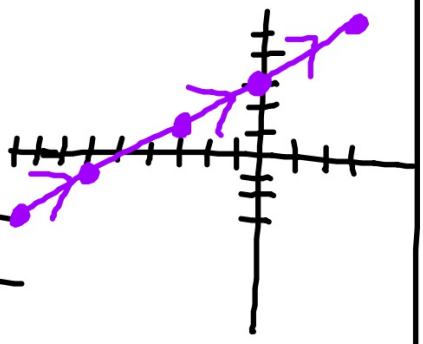
- Pg 776-777 # 3-14, 17, 18, 37-44

CW: 3-13 odd, 17, 37
HMWK: 4-14 even, 18,

$= 3t - 3$

$= 2t + 1$

x	y
-9	-3
-6	-1
-3	1
0	3
3	5



b) $x + 3 = 3t$

$\frac{1}{3}x + 1 = t$

$y = 2\left(\frac{1}{3}x + 1\right) + 1$

$y = \frac{2}{3}x + 2 + 1$

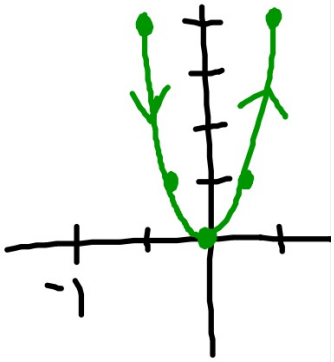
$y = \frac{2}{3}x + 3$

⑤ $x = \frac{1}{4}t$

$y = t^2$

a)

t	x	y
-2	$-\frac{1}{2}$	4
-1	$-\frac{1}{4}$	1
0	0	0
1	$\frac{1}{4}$	1
2	$\frac{1}{2}$	4



b) $4x = t$

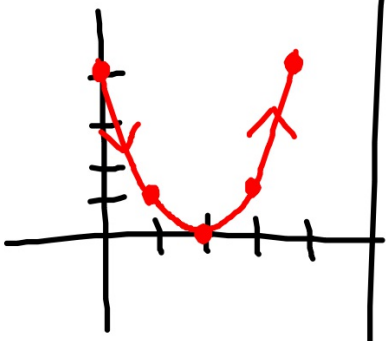
$y = (4x)^2$

$y = 16x^2$

$t+2$

$=t^2$

x	y
0	4
1	1
2	0
3	1
4	4



$x-2=t$

$y=(x-2)^2$

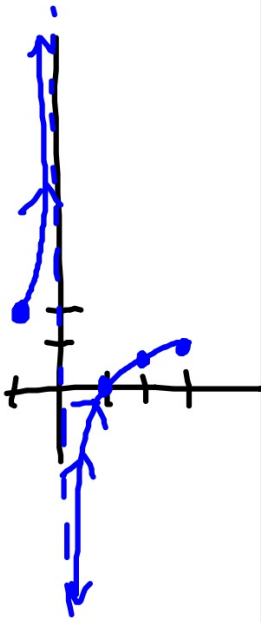
$\boxed{y=x^2-4x+4}$

⑨ $x=t+1$

$y=\frac{t}{t+1}$

a)

t	x	y
-2	-1	2
-1	0	U
0	1	0
1	2	1/2
2	3	2/3



b) $x-1=t$

$y=\frac{x-1}{x-1+1}$

$\boxed{y=\frac{x-1}{x}}$

(13) $x = 3\cos\theta$
$y = 3\sin\theta$

b) $\frac{x}{3} = \cos\theta$ / $\frac{y}{3} = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{9} = 1$

$x^2 + y^2 = 9$

$C: (0,0)$

$r = 3$

a)